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ON THE QUANTUM THEORY OF THE AUTOELECTRIC FIELD CURRENTS

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By the application of a sufficiently intense electric field a Bohr atom could be rendered unstable. For when the drop in potential across the electronic orbit reaches a value of the order of magnitude of the ionizing potential of the atom (10^9 volts/cm.), the electron, instead of remaining in the neighborhood of the nucleus will fall down the hill of potential energy, and the atom will be dissociated. This dissociation is explosive in character. For there is a critical field strength, below which the atom remains stable indefinitely and above which it dissociates in a time of the order of the orbital periods of the atom. The characteristic for the autoelectric current should accordingly show abrupt discontinuities.

If one examines the same problem with the quantum mechanics, he finds that these abrupt changes disappear: Any field, no matter how weak, will in time dissociate an atom. This is essentially a consequence of the fact that the motion of the electron is no longer absolutely restricted to a region of the dimensions of the Bohr orbit; it will now occasionally, though not very often, be found at points much further from the nucleus; and the further it is, the smaller will be the field required to insure that it does not return to the nucleus. Since the probability that an electron be at a distance R from the nucleus falls off exponentially with increasing R, the rate at which the field ionizes the atom may be expected to decrease rapidly when the field strength is decreased.

These considerations may be made precise. For, by a slight extension of the perturbation theory of the quantum mechanics, one may find the rate at which the field induces transitions; and the particular transition which is responsible for the effect is the quantum jump from the normal state of the atom to a state of the same energy, in which electron and nucleus are falling apart under the influence of the field. This rate turns out[1] to be, for a hydrogenic atom with ionizing potential W,

$$0.02\, h/m\, a^{-5/4}\, \alpha^{1/4} \exp\left\{-\tfrac{4}{3}\, a^{-3}\alpha^{-1}\right\} \qquad \begin{array}{l} a = h/2\pi(2mW)^{-1/2} \\ \alpha = 8\pi^2 meF/h^2 \end{array} \qquad (1)$$

per unit time; here m and $-e$ are the mass and charge of the electron. If the wave functions of the atom are not quite hydrogenic in character, the exponential is not essentially affected, but the numerical factor may vary by a factor, perhaps, of two.

From (1) it follows that the characteristics of the field currents should satisfy

$$ln\, i - \tfrac{1}{4}\, ln\, F = C/F. \qquad (2)$$

The reversible curves published two years ago by Millikan and Eyring[2] do, in fact, satisfy this condition very satisfactorily. And the formula (2) is essentially that found empirically and independently by Millikan and Lauritsen;[3] it has been shown by them to hold with great exactitude for all the reversible currents investigated.

With W in volts, the theoretical value of C in volts per cm. is $-10^8\, W^{3/2}$. The observed slope of the characteristic should thus make possible, provided F may be correctly estimated, the determination of the ionizing potential for electrons in metal: the work function. The values of F, however, computed from the geometrical dimensions of the wire, give for W values of a few tenths of a volt. From this we may conclude that the autoelectrons do not come uniformly from the surface of the wire, but only from certain favored points where the field strengths are abnormally high. Now this is just what is found experimentally. For the wire (after systematic out-gassing) may be subjected to fields of four million volts per centimeter without drawing a current measurable on the galvanometer;[4] but this condition may suddenly be changed,[3] so that the current jumps immediately to a large value, and one (or more) bright spots appear on the anode and the walls of the tube. If after such an event the wire is examined[4a] microscopically, it is found that at appropriate positions there are small craters, and that these are surrounded by protuberances with very small radius of curvature; and the dimensions of these points, which should be responsible for the current, vary from a tenth to a fortieth of those of the wire and thus give work functions of the order of a few volts. Thus a typical crater has a radius of curvature of 2×10^{-5} cm. and gives $W = 4.7$ volts. Diffraction, however, makes the precise estimation of the radii of curvature of the points impossible.

The formula (1) gives in amperes for the current per series electron:

$$i = 3 \times 10^{-8}\, F^{1/4}\, W^{5/8} \exp - (10^8\, W^{3/2}\, F^{-1}). \qquad (3)$$

It is thus possible, from the value of W and F, to compute the number of atoms taking part in the effect. This computation cannot be made very precise, however, because small changes in the exponential mask large changes in the number of atoms, and because, further, the value of the constant factor is not quite certain. But all the values so far obtained show that the number of atoms taking part lies between 10^{-4} and 10^{-6} of the total number of surface atoms. This explains the spotted appearance of the anode, and agrees reasonably with the dimensions of the points as observed, which give an area about 10^{-5} that of the wire.

The field currents are not absolutely independent of temperature. Empirically the characteristic curves show two distinct slopes for constant temperature and varying field; these correspond respectively to the thermionic current as modified by the field (Schottky), and to the field current as modified by the temperature; because of the steepness of the exponentials the intermediate region is not susceptible of experimental resolution. The former region is given qualitatively by Schottky's theory of the reduced image force. In the latter region one finds the law that the relative change in current induced by a change of temperature is nearly independent of the strength of the field. This law follows readily from the theory of field currents,[1] if one assumes the existence of one or more excited electronic states with work functions somewhat less than W.

Since the formula (1) was published, two theoretical treatments of the autoelectric effect have been given. The former of these, that of Houston,[5] ascribes the effect to the reduction of the work function of the metal by the field, and the internal partial pressure which the conduction electrons acquire on the Fermi statistics. This seems, however, not to lead to a satisfactory quantitative account of the experiments; for this purpose, the atom furnishes a better model than the Fermi gas. In the second paper[6] the effect is obtained, as in the present theory, from the fact that the wave functions for the electrons do not vanish completely outside the surface of the metal; but the method of computing the current is inconsistent with the quantum mechanics and involves undetermined constants which should in fact be determinate. The characteristic obtained by this calculation also fails to agree with experiment, because the binding force which holds the electron in the metal is taken to be the image force instead of the coulomb attraction of the ions, as in the present paper.

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[1] J. R. Oppenheimer, *Phys. Rev.*, **31,** 80 (1928).

[2] R. A. Millikan and C. Eyring, *Ibid.*, **27,** 55 (1926).

[3] R. A. Millikan and C. Lauritsen, *Proc. Nat. Acad. Sci.*, **14,** 45 (1928).

[4] R. A. Millikan and B. E. Shackelford, *Phys. Rev.*, **15,** 239 (1920).

[4a] I am indebted to Dr. Lauritsen for an opportunity to see his photographs of the craters.

[5] V. Houston, *Zs. Phys.*, **47,** 33 (1928).

[6] O. W. Richardson, *Roy. Soc. Proc.*, **A667,** 719 (1928).